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Research Article

Long-Memory Price Dynamics of Mustard: A Comparative Assessment of ARFIMA and ARTFIMA Models

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ABSTRACT

Accurate forecasting of agricultural commodity prices is crucial for mitigating market uncertainty and facilitating informed decision-making by farmers, traders, and policymakers, particularly in agriculture-dependent economies such as India. Mustard, a key oilseed crop, exhibits pronounced price volatility driven by factors such as climatic variability, fluctuations in input costs, and policy interventions. The primary objective of this study is to compare the forecasting performance of fractional integration-based time series models, specifically the Autoregressive Fractionally Integrated Moving Average (ARFIMA) and Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) models, for weekly mustard prices in Agra. Forecast accuracy is evaluated using the Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE). The empirical results demonstrate that the ARFIMA model outperforms the ARTFIMA model, as evidenced by lower RMSE and MAPE values. These findings indicate that long-memory behavior constitutes a dominant feature of mustard price dynamics and is more effectively captured by the ARFIMA framework. Overall, the study underscores the applicability of fractional integration models in agricultural price forecasting and provides methodological insights into model selection for commodities characterized by persistent temporal dependence.

Key words: ARFIMA, ARTFIMA, Long-Memory

Introduction

Agricultural commodity price forecasting plays a pivotal role in promoting market stability, guiding farmers' decisions, and informing policy interventions. In India, mustard is a key oilseed crop that significantly contributes to domestic consumption and supports the edible oil sector. Reliable forecasts of mustard prices enable farmers to make informed choices regarding cultivation, storage, and marketing strategies, while also providing policymakers with insights necessary to implement timely measures aimed at stabilizing markets and mitigating volatility. The accurate prediction of agricultural prices is thus essential for enhancing both economic efficiency and food security (Lütkepohl, 2013; Zivot and Wang, 2006).

Time series modeling has long been a standard approach for forecasting agricultural commodity prices. Among the most widely used methods is the Autoregressive Integrated Moving Average (ARIMA) model, which combines autoregressive (AR) and moving average (MA) components with differencing to achieve stationarity (Box and Jenkins, 1976). ARIMA models are widely appreciated for their simplicity, interpretability, and effectiveness in

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capturing short-term dependencies. However, they are limited in representing long-range dependence because they rely on integer-order differencing. Many agricultural price series, including mustard prices, exhibit persistent behavior and slowly decaying autocorrelations, characteristics that ARIMA models may fail to capture adequately (Granger and Joyeux, 1980; Hosking, 1981).

To address this limitation, the Autoregressive Fractionally Integrated Moving Average (ARFIMA) model was developed. Unlike ARIMA, ARFIMA allows the differencing parameter to assume fractional values, enabling the model to capture long-memory behavior where past observations exert influence over extended periods. This capability makes ARFIMA particularly well-suited for agricultural commodities, which often demonstrate persistent trends and gradual decay in autocorrelations. Empirical studies have applied ARFIMA to various economic and agricultural time series, demonstrating its superior forecasting performance in situations where long-term dependencies are present (Beran and Terrin, 1994; Baillie, 1996; Hyung *et al*., 2008). The flexibility of ARFIMA in modeling intermediate persistence has also made it a preferred choice in financial and macroeconomic forecasting (Diebold and Rudebusch, 1989).

Further extending the ARFIMA framework, the Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) model introduces a tempering parameter that reduces the influence of distant past observations. This modification allows the model to represent semi-long-range dependence, whereby the autocorrelation function decays slowly at shorter lags, resembling a long-memory process, but declines exponentially over longer lags, unlike the pure long-memory behavior of ARFIMA (Meerschaert *et al*., 2014; Meerschaert *et al*., 2015). This tempering mechanism improves the modeling of real-world time series, which often do not maintain persistent correlations indefinitely due to structural changes, policy interventions, or market adjustments. Recent studies have highlighted the theoretical and empirical advantages of ARTFIMA, including its stationarity under a broader range of parameters and its capacity to capture both short- and long-term dependencies in time series data (Kabala *et al*., 2021; Baeumer and Meerschaert, 2010).

The distinction between ARFIMA and ARTFIMA models is important when modeling time series with complex dependence structures. ARFIMA models represent long-memory behavior through fractional differencing, assuming that dependence persists over long horizons. ARTFIMA extends this framework by allowing the strength of dependence to diminish at longer lags, accommodating situations in which persistence weakens over time. These differing assumptions are particularly relevant for agricultural commodity prices, such as weekly mustard prices, where long-term movements may coexist with short-term variations driven by seasonality and market dynamics.

The present study aims to evaluate and compare the forecasting performance of ARFIMA and ARTFIMA models applied to weekly mustard price data from India. Forecast accuracy is assessed using widely recognized measures, including Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE). By analyzing and contrasting these two models, this study contributes to a better understanding of long-memory and tempered long-memory processes in agricultural price forecasting, providing insights for both farmers and policymakers.

Data and Methodology

Data Description

The present study utilizes a weekly price series of mustard (Rs./quintal) collected from the Agra market in India to examine and compare the forecasting performance of ARFIMA and ARTFIMA models. The data were obtained from the AGMARK database (https://agmarknet.gov.in/) maintained by the Directorate of Marketing and Inspection, Government of India. The dataset comprises 1150 weekly observations spanning the period from December 1998 to December 2020. To assess the predictive ability of the competing models and to avoid overfitting, the dataset was divided into training and testing subsets. The training dataset, comprising 1140 observations, was used for model estimation and in-sample fitting, while the remaining 10

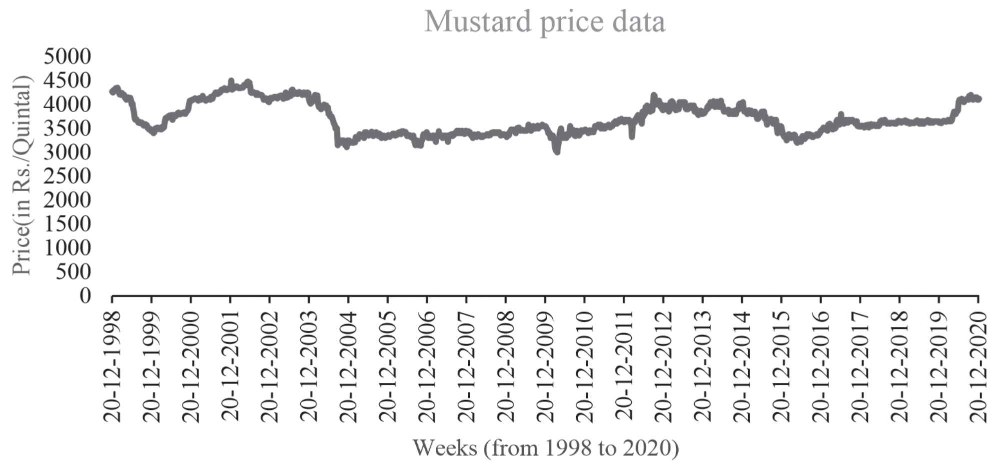

Fig.1. Graphical plot of the weekly price series data of mustard

observations were reserved as the testing dataset for out-of-sample forecasting and model validation. The use of the most recent weeks for testing allows for a realistic evaluation of forecast performance under current market conditions. All statistical analyses, model estimation, and forecasting procedures were carried out using the R statistical software.

Methodology

The orders of the ARFIMA and ARTFIMA models were determined by fitting several alternative model configurations for each model and selecting the specification that minimized the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), with parameters estimated using maximum likelihood methods (Sowell, 1992). The ARFIMA and ARTFIMA models were fitted in R using the packages *arfima* and *artfima*, respectively.

ARFIMA model

The Autoregressive Fractionally Integrated Moving Average (ARFIMA) model (Granger and Joyeux, 1980) is an advanced time series modeling framework designed to capture long-range dependence in the data (Joyeux, 2010). In contrast to the conventional ARIMA model, which employs integer-order differencing to induce stationarity, the ARFIMA model allows the differencing parameter to assume fractional values. This extension provides greater flexibility in representing persistent temporal dependence commonly observed in economic, financial, and agricultural price series. Estimation and inference for ARFIMA models have been extensively studied in the literature (Sowell, 1992; Palma, 2007), with numerous empirical applications demonstrating their relevance in practice (Cheung and Lai, 1995).

The ARFIMA(p,d,q) model is expressed as

$$\phi(B)\nabla^d Y_t = \mu + \theta(B)\varepsilon_t \tag{1}$$

where $d \in$ R represents the fractional order of integration, μ is a constant reflecting the mean of the process, ε_t is an independently and identically distributed error term with mean zero and variance σ^2, B denotes the backward shift operator, ∇^d is the fractional differencing operator, $\phi(B) = \phi_1 B - \dots - \phi_p B^p$ represents the autoregressive polynomial of order p and $\theta(B) = 1 + \theta_1 B + \dots + \theta_q B^q$ denotes the moving average polynomial of order q.

The fractional differencing operator is defined through its binomial expansion

$$\nabla^d = \sum_{j=0}^{\infty} \pi_j B^j, \tag{2}$$

where the coefficients π_j are given by

$$\pi_j = \frac{\Gamma(j-d)}{\Gamma(j+1)\Gamma(-d)}, \tag{3}$$

and $\Gamma(.)$denotes the gamma function. The polynomials $\phi(B)$ and $\theta(B)$ are assumed to share no common roots. Substituting (2) into (1) and inverting the moving average component allows the ARFIMA process to be expressed as an infinite-order autoregressive representation,

$$Y_t = \mu_0 + \sum_{j=1}^{\infty} \mu_j Y_{t-j} + \varepsilon_t, \quad (4)$$

For positive values of the process exhibits long-memory behavior, which is characterized either by a spectral density that diverges at zero frequency or by an autocorrelation function that decays hyperbolically (Martin and Wilkins, 1999). Assuming $d \in (-0.5, 0.5)$, the spectral density of the ARFIMA process is given by

$$I(\omega) = I_w(\lambda)\left[2\sin\left(\frac{\omega}{2}\right)\right]^{-2d}, \quad (5)$$

where

$$I_w(\omega) = \frac{\sigma_\varepsilon^2 \left|\theta(e^{-i\omega})\right|^2}{2\pi\left|\Phi(e^{-i\omega})\right|^2}, \quad (6)$$

corresponds to the spectral density of the associated stationary ARMA process. As the frequency ω approaches zero, the spectral density satisfies

$$I(\omega) \propto \omega^{-2d} \quad (7)$$

where the proportionality constant depends on the ARMA parameters underlying the data-generating process. Equation (7) demonstrates that the low frequency behavior of the spectrum is governed entirely by the fractional integration parameter d.

An ARFIMA(p,d,q) process $\{Y_t\}$ is said to be causal if it admits a linear representation in terms of current and past innovations:

$$Y_t = \sum_{j=0}^{\infty} a_j \varepsilon_{t-j}, \text{ where } \sum_{j=0}^{\infty} a_j^2 < \infty \text{ and } t = 0, \pm 1, \pm 2, \ldots \quad (8)$$

Similarly, the ARFIMA process is invertible if the innovation sequence can be recovered from the observations:

$$\varepsilon_t = \sum_{j=0}^{\infty} b_j Y_{t-j}, \text{ where } \sum_{j=0}^{\infty} b_j^2 < \infty. \quad (9)$$

ARTFIMA model

The Autoregressive Tempered Fractionally Integrated Moving Average (ARTFIMA) model, introduced by Meerschaert *et al.* (2014), provides an advanced framework for modeling time series exhibiting complex dependence patterns (Kumar and Chaturvedi, 2021). Let $\{Y_t\}_{t\in Z}$ enote a stochastic process. The process is said to follow an ARTFIMA(p,d,λ,q) structure if the transformed series $X_t = \nabla^{d,\lambda} Y_t = (1 - e^{-\lambda}B)^d Y_t$ satisfies an ARMA(p,q) model. Here, Y_t represents the observed time series, X_t denotes the transformed series, d is the fractional differencing parameter, $\lambda > 0$ is the tempering parameter controlling memory decay, and B is the backward shift operator.

The tempered fractional differencing operator is defined as

$$\Delta^{d,\lambda} f(t) = \left(1 - e^{-\lambda}B\right)^d f(t) = \sum_{j=0}^{\infty} \eta_j^{d,\lambda} f(t-j) \quad (10)$$

where d > 0, $d \notin \mathbb{Z}$, $\lambda > 0$, $Bf(t) = f(t-1)$. The coefficients$\eta_j^{d,\lambda}$ are given by

$$\eta_j^{d,\lambda} = (-1)^j \binom{d}{j} e^{-\lambda j}, \quad (11)$$

with the generalized binomial coefficient defined as $\binom{d}{j} = \frac{\Gamma(1+d)}{j!\Gamma(1+d-j)}$.

The ARMA(p,q) component governing the transformed process X_t is specified by

$$X_t - \sum_{j=1}^{p} \phi_j X_{t-j} = \varepsilon_t + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i}, \quad (12)$$

where $\{\varepsilon_t\}_{t\in Z}$ is a white noise sequence with zero mean and constant variance. The autoregressive and moving average polynomials are defined as $\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$ and $\theta(B) = 1 + \theta_1 B + \ldots \theta_q B^q$.

For an ARTFIMA(p,d,λ,q) process satisfying $|\phi(z)| > 0$ and $|\theta(z)| > 0$, the series $\{Y_t\}$ admits a causal representation $Y_t = \sum_{j=0}^{\infty} a_j^{-d,\lambda} \varepsilon_{t-j}$, with $\sum_{j=0}^{\infty}\left|a_j^{-d,\lambda}\right| < \infty$. Additionally, it is invertible, given by $\varepsilon_t = \sum_{j=0}^{\infty} c_j^{d,\lambda} Y_{t-j}$, where $\sum_{j=0}^{\infty}\left|c_j^{d,\lambda}\right| < \infty$.

The ARFIMA(p,d,q) model emerges as a special case of the ARTFIMA framework when the tempering parameter $\lambda = 0$. While ARFIMA models are effective in capturing long-range dependence through the fractional differencing parameter d, they are characterized by a non-summable covariance function, which can complicate statistical inference. The ARTFIMA model overcomes this limitation by incorporating the tempering parameter λ, yielding a summable covariance structure that facilitates both theoretical analysis and practical implementation. An important feature of the ARTFIMA model is its ability to represent semi-long-range dependence. Specifically, the autocovariance function initially behaves like that of a long-memory process, decaying slowly over early lags, but ultimately declines at an exponential rate as the lag increases (Sabzikar *et al.*, 2015). Consequently, correlations between distant observations weaken more rapidly than in a pure long-memory ARFIMA process. Unlike ARFIMA models, which are stationary only when -0.5 < d < 0.5, the ARTFIMA model remains stationary for any $d \notin \mathbb{Z}$ provided $\lambda > 0$, as the tempering mechanism effectively mitigates unit root behavior (Sabzikar *et al.*, 2019; Kabala *et al.*, 2021).

Accuracy Measures

Root Mean Squared Error (RMSE) measures the forecast errors by taking the square root of the mean of squared deviations between the observed and predicted values. It is defined as:

$$RMSE = \sqrt{\frac{1}{h}\sum_{s=1}^{h}(y_s - \hat{y}_s)^2} \tag{13}$$

where y_s denotes the actual observed value at time s, y_s, represents the corresponding forecasted value, and h is the forecast horizon. Lower RMSE values indicate better forecasting accuracy.

Mean Absolute Percentage Error (MAPE) expresses forecast accuracy as a percentage and measures the average absolute deviation of forecasts from actual values relative to the observed data. It is given by:

$$MAPE = \frac{1}{h}\sum_{s=1}^{h}\left|\frac{y_s - \hat{y}_s}{y_s}\right| \times 100 \tag{14}$$

Results and Discussion

The weekly mustard price series for the Agra market was first examined using descriptive statistical measures to gain preliminary insights into its distributional characteristics. Table 1 summarized the key descriptive statistics for the period 1998-2020. The dataset consisted of 1,150 weekly observations with an average price of Rs. 3694.65 and a standard deviation of Rs. 324.36, indicating moderate variability over time. The positive skewness value (0.45) suggested a slight right-skew in the distribution, while the negative kurtosis (–0.90) reflected a relatively flatter distribution compared to normality. These features are typical of agricultural commodity prices, which were often influenced by seasonal patterns, production shocks, and market uncertainty.

Table 1. Descriptive Statistics of weekly mustard price series (1998-2020)

Statistics	Agra market
Observations	1150
Mean (Rs.)	3694.65
Median (Rs.)	3630
Maximum (Rs.)	4500
Minimum (Rs.)	3000
Standard Deviation (Rs.)	324.36
Skewness	0.45
Kurtosis	0.90

Figures 2 and 3 displayed the autocorrelation function (ACF) and partial autocorrelation function (PACF) plots of the weekly mustard price series, respectively. The ACF exhibited a slow and gradual decay across increasing lags, indicating strong persistence and the presence of long-range dependence in the series. In contrast, the PACF showed a prominent spike at the first lag followed by relatively small values at higher lags, suggesting a low-order autoregressive structure combined with

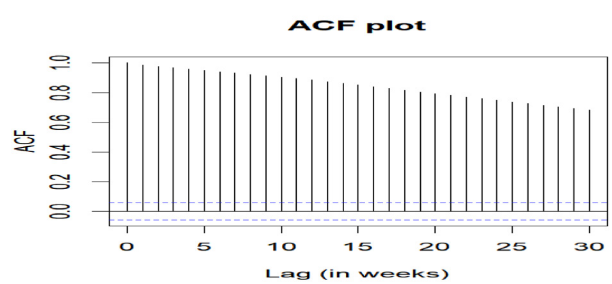

Fig. 2. ACF plot of weekly mustard prices (1998–2020)

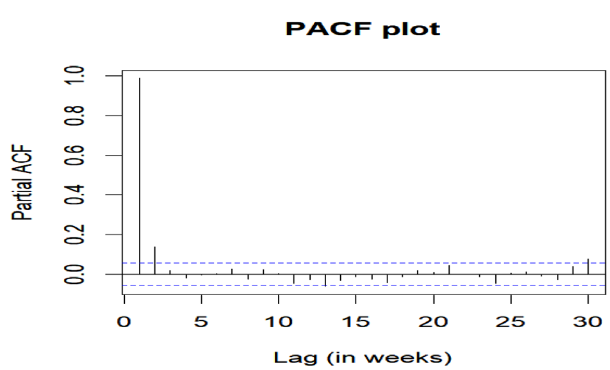

Fig. 3. PACF plot of weekly mustard prices (1998-2020)

Table 2. Long-memory test results for weekly mustard price series (1998-2020)

Test Method	Parameter	Estimate
GPH Test	d (Fractional differencing parameter)	0.32
R/S Analysis	H (Hurst exponent)	0.71

long-memory behavior. These patterns provided preliminary support for the use of fractional integration-based models such as ARFIMA and ARTFIMA.

To assess the presence of long-range dependence in the mustard price series, formal long-memory tests (Pilar Grau-Carles, 2005) were conducted using the Geweke-Porter-Hudak (GPH) method (Geweke and Porter Hudak, 1983) and rescaled range (R/S) analysis (G C Nath, 2001). The results, reported in Table 2, provided strong evidence of persistent dependence. The GPH test yielded a fractional differencing parameter estimate of d = 0.32, indicating significant long-memory behavior. Consistent with this finding, the R/S analysis produced a Hurst exponent value of 0.71, which exceeded the benchmark value of 0.5 associated with random walk behavior. Together, these results confirmed the presence of long-range dependence in weekly mustard prices and justified the application of fractional integration-based models.

Based on these diagnostic findings, ARFIMA and ARTFIMA models were estimated using the logarithmically transformed series to stabilize variance and improve estimation efficiency. The estimated model specifications, parameter values, and in-sample information criteria were presented in Table 3. Both models were fitted with AR(1) and MA(1) components, while differing in their treatment of persistence. The ARFIMA model captured long-memory through fractional differencing, with a fractional differencing parameter of d 0.03. In contrast, the ARTFIMA model incorporated an additional tempering parameter, $\lambda = 0.02$, along with a fractional differencing parameter of d 1.81. The AR coefficients were 0.99 and 0.04, and the MA coefficients were 0.19 and 0.91 for the ARFIMA and ARTFIMA models, respectively. In terms of in-sample fit, the ARTFIMA model achieved slightly lower AIC and BIC values than the ARFIMA model, indicating a marginally better representation of the training data. However, this improvement came at the cost of increased model complexity, which might affect forecasting performance.

Table 3. Parameter estimates and information criteria for ARFIMA and ARTFIMA models

Metric	ARFIMA(p,d,q) model	ARTFIMA($p,d,,q$) model
Model order	(1,0.03,1)	(1,1.81,0.02,1)
AR coefficient	0.99	0.04
MA coefficient	0.19	0.91
AIC(Train)	12000.76	12003.98
BIC(Train)	12020.95	12034.20

The out-of-sample forecasting ability of the two models was evaluated using the last 10 weeks of observations, with forecast accuracy assessed through Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE). The results were reported in Table 4. The ARFIMA model achieved lower error measures, with an RMSE of 30.08 and a MAPE of 0.62, compared to the ARTFIMA model, which recorded an RMSE of 33.32 and a MAPE of 0.70. These results suggested that the ARFIMA model provided more accurate forecasts for the mustard price series in the evaluation period.

Table 4. Out-of-sample forecast accuracy measures for ARFIMA and ARTFIMA models

Model	RMSE	MAPE
ARFIMA	30.08	0.62
ARTFIMA	33.32	0.70

Table 5 summarized the actual weekly mustard prices together with the point forecasts and corresponding 95% prediction intervals obtained from the ARFIMA and ARTFIMA models for the final ten weeks of 2020. For each forecasted week, the table reported the observed price, the estimated forecast value, and the associated lower and upper prediction bounds, thereby offering a clear representation of forecast uncertainty. The prediction intervals provided insight into the degree of uncertainty surrounding the point forecasts and displayed a modest widening as the forecast horizon

Table 5. Forecasts and 95% Prediction Intervals for Weekly Mustard Prices: Last 10 Weeks of 2020

Week	Actual	ARFIMA Forecast	ARFIMA Lower 95	ARFIMA Upper 95	ARTFIMA Forecast	ARTFIMA Lower 95	ARTFIMA Upper 95
1	4200	4144.73	4075.71	4214.92	4142.21	4036.89	4250.28
2	4150	4139.53	4042.87	4238.50	4135.28	3987.37	4288.68
3	4120	4134.39	4017.07	4255.13	4129.01	3948.86	4317.38
4	4100	4129.31	3994.99	4268.13	4123.23	3916.21	4341.19
5	4150	4124.28	3975.35	4278.80	4117.82	3887.37	4361.93
6	4140	4119.32	3957.47	4287.79	4112.71	3861.27	4380.54
7	4150	4114.41	3940.95	4295.50	4107.86	3837.26	4397.54
8	4130	4109.56	3925.54	4302.21	4103.22	3814.93	4413.29
9	4150	4104.77	3911.05	4308.08	4098.76	3793.99	4428.02
10	4100	4100.03	3897.34	4313.26	4094.47	3774.21	4441.91

increased, which aligned with established principles of time series forecasting. A comparison between the observed prices and the prediction intervals indicated that the majority of actual values fell within the 95% prediction bands produced by both models. This suggested that the forecasts were generally consistent with the observed market outcomes.

Figure 4 presented a comparison between the actual weekly mustard prices and the forecasts obtained from the ARFIMA and ARTFIMA models for the last 10 weeks of 2020. The actual price series exhibited noticeable fluctuations, highlighting the inherent volatility of the mustard market during the study period. The ARFIMA forecasts remained relatively closer to the actual price movements across most of the weeks, effectively capturing the underlying price dynamics over time. The ARTFIMA model, while still capturing the overall trend, showed slightly larger deviations in most of the weeks, which could be attributed to its semi-long-range dependence property: it initially behaved like a long-memory process but the correlations decayed exponentially for distant observations. Neither model fully captured sharp deviations observed in weeks 2, 5, and 7, where the actual prices showed sudden increases or decreases. Nevertheless, the ARFIMA model demonstrated better alignment with the observed series across most weeks, indicating its superior forecasting performance.

Overall, the empirical results suggested that weekly mustard prices were characterized by strong and persistent long-memory behavior. Although the ARTFIMA model offered a flexible framework and achieved marginally better in-sample fit, this advantage did not translate into improved forecasting accuracy. The ARFIMA model, by directly modeling

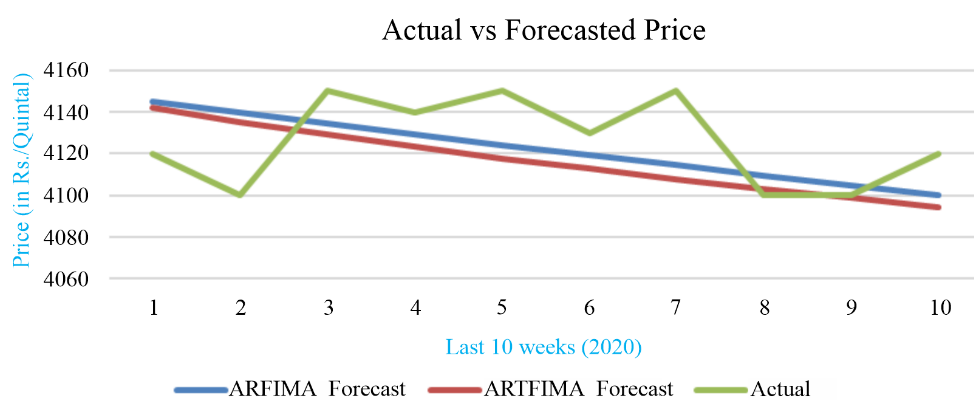

Fig. 4. Comparison of actual and forecasted prices (by ARFIMA and ARTFIMA models) for Agra mustard price series (last 10 weeks of 2020)

persistent dependence with a parsimonious structure, was found to be more effective for forecasting in this application. These findings highlighted the importance of selecting forecasting models that align with the underlying dependency structure of agricultural price series.

Conclusion

This study conducted a comparative assessment of ARFIMA and ARTFIMA models for forecasting weekly mustard prices in the Agra market, with particular emphasis on modeling long-range dependence in the price series. The empirical analysis confirmed the presence of strong and persistent long-memory characteristics in mustard prices over the study period. Consistent with these properties, the ARFIMA model demonstrated superior out-of-sample forecasting performance, as reflected by lower RMSE and MAPE values compared to the ARTFIMA model. Although the ARTFIMA model offered additional flexibility by incorporating a tempering parameter to account for semi-long-range dependence, this feature did not translate into improved forecasting accuracy for the mustard price series considered in this study. The results suggested that the dominant dependence structure in weekly mustard prices was better represented by persistent long-memory behavior rather than tempered dependence, particularly over short forecasting horizons. Graphical comparisons of actual and predicted prices further supported this finding, as ARFIMA forecasts generally tracked observed price movements more closely across most forecasted weeks.

While both models exhibited limitations in capturing abrupt price fluctuations during certain weeks, the overall results highlighted the effectiveness of the ARFIMA framework in modeling and forecasting agricultural commodity prices characterized by strong persistence. These findings emphasized the importance of aligning model selection with the underlying dependence structure of the data, rather than relying solely on model flexibility or in-sample fit. Future research may extend this analysis by incorporating exogenous variables such as weather indicators or policy interventions. Such approaches could further enhance forecasting accuracy and support better decision-making for farmers, traders, and policymakers in agricultural markets.

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